

ZADIE Functional Thinking through Patterning

Early algebra teacher preparation programme

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Note: Please contact Aisling Twohill (aisling.twohill@dcu.ie) if you would like access to the original slides and/or would be interested in a cross-case study between your research and research we have conducted using these resources.

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Session 1

Functions
Ways of thinking about functions
Ways of representing functions

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Functions

“any expression made up of a variable and some constants” (Johann Bernoulli, 1667-1748)

“A function may be thought of a rule, or correspondence, that associates with element of a set X one and only one element of a set Y .” (Swokowski, 1979)

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Describe the pattern and find a rule.

1 2 3 4

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Ways of thinking about functions

Recursive: Exploring and finding variation within a pattern by relating the quantity of squares from one figure to another

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Ways of thinking about functions

Co-variational thinking: Analysing how two quantities, i.e. the position number (ordinal) and the quantity (cardinal), vary simultaneously.

Usually children encounter ordinal numbers that are labels. Such labels are not "worked with". In patterning, we expect children to see these ordinal numbers as inputs and this may cause a challenge for some children.

The increase of the position (independent variable) by one causes an increase by three in the quantity (dependent variable). The co-variation describes the *change* that works for each and every relation between the independent and dependent variable in the specific pattern.

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Ways of thinking about functions

The constant: Identifying the change needs to go hand in hand with identifying constant elements. Spatial patterns allow for using the spatial structure to verbally see the constant part and mark them.

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Ways of thinking about functions

Explicit: describing the structure of the pattern by connecting the position number to the number of squares.

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Ways of representing functions

What have we used so far?

- Visual (pattern)
- Verbal
 - Informal – describe in words
 - Formal Rule – using symbols

What other ways could we represent functions?

- Tables; graphs; contexts

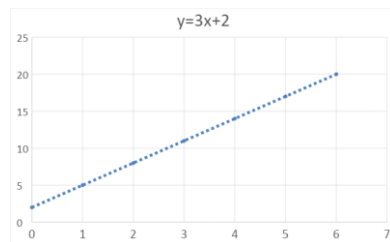
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Ways of representing functions: tables

1	2	3	4	...	n
$1 + 3 \times 1 + 1$	$1 + 3 \times 2 + 1$	$1 + 3 \times 3 + 1$	$1 + 3 \times 4 + 1$		$1 + 3 \times n + 1$
$3 \times 1 + 2$	$3 \times 2 + 2$	$3 \times 3 + 2$	$3 \times 4 + 2$		$3 \times n + 2$
	3 times the two and two more	...			3 times the position number and two more

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Ways of representing functions: graphs



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Ways of representing functions: context

You have 2 coins in your money box, and each week you save 3 more.

1	2	3	4	...	n
$2 + 1 \times 3$	$2 + 2 \times 3$	$2 + 3 \times 3$	$2 + 4 \times 3$		$2 + n \times 3$
	2 times the three and two more	...			The position number times 3 and two more

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Key Ideas

- Functions
- Ways of thinking about functions:
 - Recursive
 - Covariation
 - Explicit (a constant and a rate of change)
- Ways of representing functions
 - Visual pattern
 - Verbal
 - Function rule
 - Table (as a tool to represent structure)
 - Context
 - Graph

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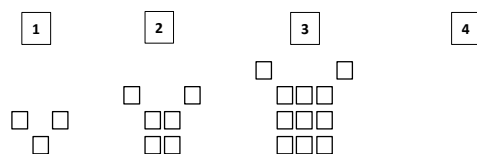
Session 2

Linear and non-linear functions
 Tables and over generalising recursive approaches
 Seeing structure within a pattern in different ways
 Function rules as expressions of different perspectives of structure
 Comparing expressions for equivalence
 Generalisation (linguistic forms)

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Activity 1



Describe the pattern using any representation. Apply the ways of thinking about functions from Preparation Session 1.

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Activity 1: Maria's thinking

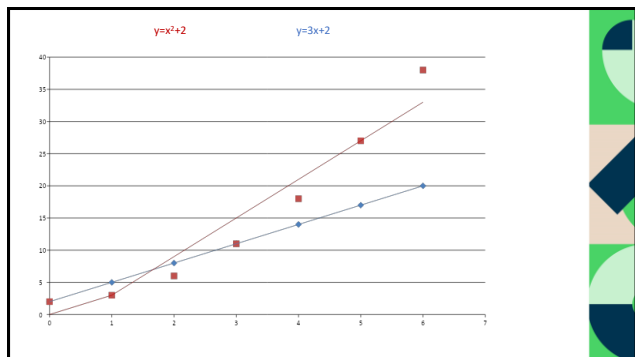
Maria says "I made this table and I noticed the difference is increasing in twos"

Figure no	1	2	3	4	5
Number of squares	3	6	11	18	27

$\underbrace{\quad\quad\quad}_{+3}$
 $\underbrace{\quad\quad\quad}_{+5}$
 $\underbrace{\quad\quad\quad}_{+7}$
 $\underbrace{\quad\quad\quad}_{+9}$

Is Maria's thinking helpful? Can she use her strategy to describe figure 50 in the pattern?

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Seeing in different ways.

In how many different ways can you describe the pattern? Indicate by using different colours.

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How would each child describe figure 10?

Aisling

Anna

Sharon

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How would each child describe any figure in the pattern?

Aisling

Anna

Sharon

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Generalisation

Generalisation describes the pattern structure in a way that is true for all figures in this pattern.

For this pattern you have:

- Expressed generalisations in words
- Expressed generalisations using quasi-variables
- Expressed generalisations using abstract symbols (function rule)

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- ### Key Ideas
- Linear and non-linear functions
 - Tables and over generalising recursive approaches
 - Seeing structure within a pattern in different ways
 - Function rules as expressions of different perspectives of structure
 - Comparing expressions for equivalence
 - Generalisation (linguistic forms)

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Session 3

Functional thinking
 Typical patterning tasks
 Comparing patterns and comparing functions
 Analysing and working with children's responses
 Interrogating resources

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Functional Thinking

- Identifying structure and describing relationships and change.
- "representational thinking that focuses on the relationship between two (or more) varying quantities, specifically the kinds of thinking that lead from specific relationships (individual incidences) to generalisations of that relationship across instances" (Smith, 2008)
- "Functional thinking relates to understanding the notion of change and how varying quantities (or variables) relate to one another" (Wilkie, 2015).

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Typical patterning tasks

Display some terms of a geometric pattern:

1 2 3 4

This is a scaffolded structure. As children develop proficiency in solving patterns, it would be appropriate to remove interim steps and ask them to find the function rule. In this way you are anticipating that they have internalised an approach.

Ask children to:

- Describe the pattern;
- Find (draw/construct) the next figure;
- Describe/find a near figure;
- Describe/find a far figure.

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Find the function rule

1 2 3

1 2 3

Find the function rule for both patterns.
 Compare your findings. Does anything strike you?

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Find the function rule

1 2 3 4

1 2 3 4

Find the function rule for both patterns. Compare your findings. Does anything strike you?

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Find the function rule

1 2 3

1 2 3

Find the function rule for both patterns. Compare your findings. Does anything strike you?

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Greg's thinking

Greg circled the twos and said:

- 1: 3×2
- 2: 4×2
- 3: 5×2
- 4: 6×2
- n: $(n+2) \times 2$

How is Greg's approach different to approaches we have explored up to now? What will you say to Greg?

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Children's responses

Figure Number	1	2	3	4	5	...	50
Number of squares	3	5	7	9	11	...	110

Heidi completed the above table. Describe her thinking. What would you propose would be a suitable question or activity for Heidi to support her developing functional thinking?

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Children's responses

Figure Number	1	2	3	4	5	...	n
Number of squares	3	5	7	9	11	...	$n+2$

Adam completed the above table. Describe his thinking. What would you propose would be a suitable question or activity for Adam to support his developing functional thinking?

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Interrogating resources

2. Extend the geometric pattern and complete the table. You may need extra paper for C and D.

a.

Pattern	1	2	3	4	5	6	10
Number of blocks							

b.

Pattern	1	2	3	4	5	6	10
Number of blocks							

What do you think? How does this develop/hinder functional thinking?

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Key Ideas

- Functional thinking
- Typical patterning tasks
- Comparing patterns and comparing functions
 - Same rule, different patterns
 - Same slope, different constant
 - Same constant, different slope
- Analysing and working with children's responses
- Interrogating resources

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